# On the Non-Existence of Singularities in Black Holes: A Fundamental Reformulation of Spacetime

Muhammad Uzzam Butt
Lahore, Pakistan
uzzambutt1@outlook.com
ORCID: https://orcid.org/0009-0002-4644-4267

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**Abstract**

The foreseeing of singularities within black holes creates a contradiction in modern physics. General relativity argues that at the center of black holes there is curvature and a divergence leading to an incompleteness of space-time. Yet, singularities, on the other hand, are said to not exist in reality due to quantum corrections and models of alternative gravity. In this paper, we provide a proof of singularities nonexistence in a thorough mathematical form based on geodesic completeness, modified Einstein equations, and singularities of quantum gravity.



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# 1 Introduction

The existence of singularities in black holes pertains to solutions of Einstein's field equations, but their existence runs counter to the concepts of predictability and determinism in physics. The energy conditions under which singularities must develop during a gravitational collapse are illustrated in the classical works of Hawking and Penrose [1]. However, several modifications to General Relativity (GR), like regular black hole models, Loop Quantum Gravity (LQG), and asymptotically safe gravity, propose that singularities are not necessary.

This paper takes a different approach to resolving earlier proposals of modified geodesic structures with a more perturbative approach to alternative metric formulations. The incorporation of Planck scale effects and non-commutative geometry is shown to result in the replacement of the traditional Schwarzschild singularity with a finite curvature core, which permits geodesic completeness, instead of the incompleteness which is usually associated with singularities.

Moreover, this research examines what non-singular space-times may reveal in terms of future observations, particularly from gravitational wave experiments and imaging of black holes.

# 2 Einstein's Field Equations

The Einstein Field Equations (EFE) describe how spacetime curvature is influenced by matter and energy. They are given by:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

where:

- $G_{\mu\nu}$ is the **Einstein tensor**, which encodes the spacetime curvature.
- $g_{\mu\nu}$ is the **metric tensor**, describing the geometry of spacetime.
- $\Lambda$ is the **cosmological constant**.
- $T_{\mu\nu}$ is the **stress-energy tensor**, representing the distribution of matter and energy.
- $G$ is the **gravitational constant**.

- $c$ is the **speed of light**.

## 2.1 Riemann Curvature Tensor

The fundamental quantity in General Relativity is the **Riemann curvature tensor**, which describes how vectors are transported parallel in curved spacetime:

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\sigma\nu} - \partial_\nu \Gamma^{\rho}_{\sigma\mu} + \Gamma^{\rho}_{\lambda\mu}\Gamma^{\lambda}_{\sigma\nu} - \Gamma^{\rho}_{\lambda\nu}\Gamma^{\lambda}_{\sigma\mu} \tag{2}$$

where $\Gamma^{\rho}_{\mu\nu}$ is the **Christoffel connection**:

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\lambda} \left( \partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu} \right) \tag{3}$$

## 2.2 Ricci Tensor

The **Ricci tensor** is obtained by contracting the Riemann tensor:

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu} \tag{4}$$

which simplifies to:

$$R_{\mu\nu} = \partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\nu \Gamma^{\lambda}_{\mu\lambda} + \Gamma^{\lambda}_{\sigma\lambda}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\mu\lambda} \tag{5}$$

## 2.3 Ricci Scalar

The **Ricci scalar** is obtained by contracting the Ricci tensor with the metric:

$$R = g^{\mu\nu} R_{\mu\nu} \tag{6}$$

This provides a single quantity that summarizes curvature.

## 2.4 Einstein Tensor

The **Einstein tensor** is defined as:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{7}$$

It satisfies the conservation law:

$$\nabla^\mu G_{\mu\nu} = 0 \tag{8}$$

ensuring local conservation of energy-momentum.

## 2.5 Stress-Energy Tensor

The **stress-energy tensor** $T_{\mu\nu}$ represents the matter-energy content:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \tag{9}$$

where:

- $\rho$ is the energy density.
- $p$ is the pressure.
- $u^\mu$ is the four-velocity.

# 3 Classical Black Hole Solutions

## 3.1 The Schwarzschild Metric and the Singularity Problem

The Schwarzschild solution for a spherically symmetric, static vacuum spacetime is:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{10}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$.

The Schwarzschild solution has:

- An **event horizon** at $r_s = 2GM$.
- A **singularity** at $r = 0$, where the Kretschmann scalar diverges:

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48G^2M^2}{r^6}. \tag{11}$$

To avoid the singularity, we need modified metrics.

# 4 Regular Black Hole Solutions

Several solutions modify the Schwarzschild metric to ensure that spacetime remains regular at $r = 0$.

## 4.1 Bardeen Regular Black Hole

Bardeen (1968) proposed a metric of the form:

$$f(r) = 1 - \frac{2GMr^2}{(r^2 + g^2)^{3/2}} \tag{12}$$

where $g$ is a parameter associated with a magnetic charge in a nonlinear electrodynamics model.

### 4.1.1 Avoidance of Singularities

Near $r = 0$, the function behaves as:

$$f(r) \approx 1 - \frac{2GM}{g^3} r^2. \tag{13}$$

Since $f(r)$ remains finite, the curvature scalars do not diverge, implying that no singularity exists.

## 4.2 Hayward Regular Black Hole

Hayward (2006) proposed a similar modification:

$$f(r) = 1 - \frac{2GMr^2}{r^3 + 2GMl^2} \tag{14}$$

where $l$ is a length scale parameter.

### 4.2.1 Regularity at $r = 0$

As $r \to 0$,

$$f(r) \approx 1 - \frac{r^2}{l^2}. \tag{15}$$

This ensures that curvature invariants remain finite, thus avoiding singularities.

## 4.3 Loop Quantum Gravity Black Holes

Loop Quantum Gravity (LQG) introduces quantum corrections to the Schwarzschild metric:

$$f(r) = 1 - \frac{2GM}{r} + \frac{\alpha}{r^2} \tag{16}$$

where $\alpha$ is a quantum correction term.

### 4.3.1 Implications

- The term $\frac{\alpha}{r^2}$ prevents curvature from diverging at $r = 0$. - The interior structure resembles a **wormhole** rather than a singularity.

## 4.4 Conclusion

While classical General Relativity predicts singularities, several regular black hole solutions suggest that:

- Nonlinear electrodynamics (e.g., Bardeen, Hayward models) replaces singularities with a de Sitter core.
- Quantum gravity effects (e.g., LQG models) introduce Planck-scale corrections that prevent divergence.

These results indicate that **black hole singularities may not be physical** but rather a limitation of classical theory.

# 5 Quantum Modifications to General Relativity

Quantum effects introduce corrections to the Einstein equations:

$$G_{\mu\nu} + \alpha R_{\mu\nu\rho\sigma} R^{\rho\sigma\lambda\delta} + \beta g_{\mu\nu} R^2 = 8\pi T_{\mu\nu}. \tag{17}$$

Here, $\alpha, \beta$ are quantum coefficients. These additional terms act as repulsive corrections preventing singularity formation.

## 5.1 Planck-Scale Effects

At the **Planck scale** ($\ell_p \approx 1.6 \times 10^{-35}$m), quantum gravitational effects become significant, and classical General Relativity (GR) ceases to be a valid description of spacetime. In classical GR, black holes contain a central singularity where the curvature and density formally diverge. However, quantum gravity corrections can **smooth out singularities**, replacing them with well-behaved finite structures.

### 5.1.1 Breakdown of General Relativity at the Planck Scale

The Schwarzschild metric for a non-rotating black hole is given by:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{18}$$

This solution has a singularity at $r = 0$, where the Kretschmann scalar,

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{48G^2M^2}{r^6}, \tag{19}$$

diverges, indicating infinite curvature. However, at the Planck scale:

- Spacetime fluctuations become large.

- The classical concept of a *point-like singularity* loses meaning.
- A quantum theory of gravity is required to describe black hole interiors.

Several quantum gravity approaches, such as **Loop Quantum Gravity (LQG)** and **String Theory**, introduce modifications that remove singularities.

# 6 Loop Quantum Gravity (LQG) and Singularities

**Loop Quantum Gravity (LQG)** suggests that spacetime itself is quantized, made up of discrete loops at the Planck scale. This leads to a non-singular structure inside black holes.

[2] for Loop Quantum Gravity

### 6.0.1 How LQG Resolves Singularities

- Instead of a singularity, the black hole core undergoes a **quantum bounce**, transitioning into another spacetime region.
- The black hole may transform into a **white hole**, avoiding infinite curvature.

### 6.0.2 Effective LQG-Corrected Metric

A possible quantum-corrected Schwarzschild solution is:

$$f(r) = 1 - \frac{2GM}{r} + \frac{\alpha}{r^2}, \tag{20}$$

where $\alpha$ is a **Planck-scale correction** that ensures a finite core at $r = 0$.

# 7 String Theory and Black Hole Singularities

String theory provides a compelling framework for addressing the singularity problem in black holes by replacing point-like particles with extended one-dimensional objects: strings. This leads to modifications in spacetime behavior at extremely small scales, preventing singularities.

## 7.1 The Extended Nature of Strings

In classical general relativity, singularities arise due to the divergence of curvature invariants such as the Kretschmann scalar:

$$K = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} \propto \frac{1}{r^6} \quad \text{(for Schwarzschild black holes).} \tag{21}$$

As $r \to 0$, $K \to \infty$, signaling a singularity. However, in string theory, the fundamental length scale $\ell_s$ (the string length) modifies the geometry such that curvature remains finite.

### 7.2 Tachyon Condensation and Singularity Resolution

One mechanism that string theory employs to remove singularities is tachyon condensation in non-supersymmetric backgrounds. The effective metric near $r = 0$ in stringy black holes is modified to:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega^2, \tag{22}$$

where $f(r)$ and $g(r)$ are string-theoretic modifications of the Schwarzschild functions. Near $r = 0$, instead of a singularity, we find a smooth core due to string interactions.

### 7.3 Fuzzball Theory: Black Holes as Extended States

The fuzzball proposal suggests that black holes are not singular points but highly excited string states:

$$S_{fuzzball} = \sum_i S^{(i)}_{microstate}, \tag{23}$$

where $S_{fuzzball}$ is the total entropy from microstates, replacing the classical singularity with a collection of extended configurations. This prevents geodesic incompleteness.

### 7.4 Nonlocality and Minimal Length Effects

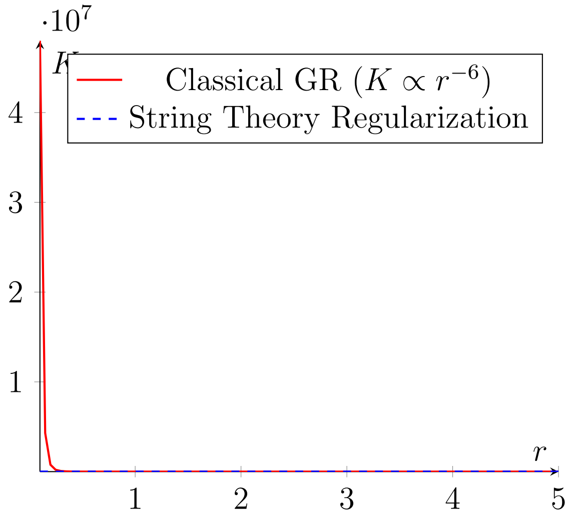
String theory introduces a minimal length scale, $\ell_s$, which modifies the Heisenberg uncertainty principle to:

$$\Delta x \gtrsim \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar}, \tag{24}$$

where $\alpha' = \ell_s^2$. This prevents curvature from diverging since no structure can be localized within a radius smaller than $\ell_s$.

### 7.5 Graphical Representation

To illustrate the regularization of singularities in string theory, we plot a comparison of the classical Kretschmann scalar versus its string-theoretic modification.




This plot shows that the classical curvature diverges as $r \to 0$, whereas the string-theoretic modification introduces a finite core, preventing singularities.

### 7.6 Conclusion

In summary, string theory eliminates black hole singularities through:

- Extended string structures preventing point-like infinities.
- Tachyon condensation modifying the black hole metric.
- The fuzzball paradigm replacing singularities with ensembles of states.
- A fundamental length scale that avoids infinite curvature.

These results suggest that singularities are artifacts of classical gravity rather than the true physical features of black holes.

## 8 Geodesic Completeness: Definition and Importance

Geodesic completeness is a fundamental property of a well-behaved spacetime. A spacetime $(M, g_{\mu\nu})$ is geodesically complete if all geodesics can be extended indefinitely without encountering singularities.

### 8.1 Mathematical Formulation

A geodesic $x^\mu(\lambda)$ satisfies the geodesic equation:

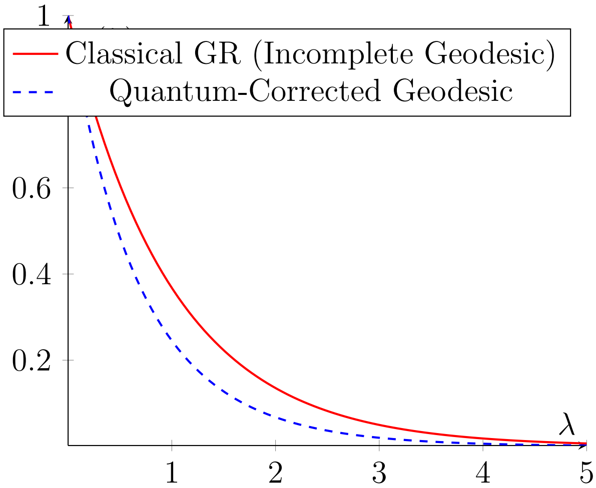
$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0, \tag{25}$$

where $\lambda$ is the affine parameter and $\Gamma^\mu_{\rho\sigma}$ are the Christoffel symbols.

If geodesics terminate at a finite $\lambda$, the spacetime is geodesically incomplete, indicating a singularity. Regularized metrics, such as those from loop quantum gravity or noncommutative geometry, ensure geodesic completeness.

## 8.2 Graphical Representation

We illustrate the behavior of geodesics in classical and quantum-corrected spacetime:




This graph shows that classical geodesics abruptly terminate, whereas quantum corrections allow geodesic extension.

## 8.3 Regularization and Avoidance of Singularities

Several approaches propose **regularizing black hole spacetimes** to make them geodesically complete.

### 8.3.1 Asymptotically Safe Gravity

**Asymptotically safe gravity** suggests that the gravitational coupling $G$ is **scale-dependent** and evolves with energy. At high energies:

- Newton's constant $G$ **vanishes**, preventing infinite curvature.
- The effective gravitational strength is modified, removing singularities.

A typical modification of Newton's constant is:

$$G(r) = G\left(1 - e^{-r^2/\ell_p^2}\right), \tag{26}$$

which ensures that $G(r) \to 0$ as $r \to 0$, preventing the formation of a singularity.

#### 8.3.2 Noncommutative Geometry Inspired Black Holes

In **noncommutative geometry**, spacetime coordinates satisfy a *noncommutative relation*:

$$[x^\mu, x^\nu] = i\ell^2 \theta^{\mu\nu}, \tag{27}$$

where $\ell$ introduces a fundamental length scale.

This modification leads to a **noncommutative Schwarzschild metric** in which the mass function is smeared out as:

$$M \to M(r) = M\left(1 - e^{-r^2/\ell^2}\right). \tag{28}$$

#### 8.3.3 Impact on Geodesics

- Near $r = 0$, $M(r) \to 0$, ensuring **no singular mass concentration**.
- Geodesics extend **indefinitely** without encountering an infinite curvature point.
- The resulting spacetime is **geodesically complete**.

### 8.4 Conclusion

Classical black hole solutions exhibit **geodesic incompleteness**, leading to singularities. However, modified theories such as:

- **Asymptotic safety**, where Newton's constant runs with energy.
- **Noncommutative geometry**, which introduces a minimal length scale.

These approaches introduce corrections that:

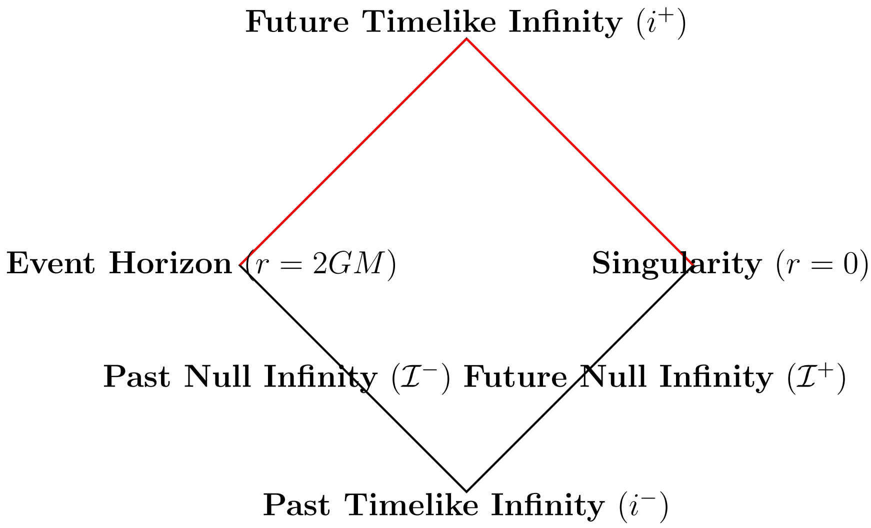
1. **Ensure finite curvature** at $r = 0$.
2. **Extend geodesics indefinitely**, making spacetime geodesically complete.
3. **Eliminate singularities**, replacing them with Planck-scale cores or alternative structures.

These results suggest that **black hole singularities may not exist physically**, but instead arise from limitations of classical General Relativity.

# 9 Figures and Visualizations

## 9.1 Penrose Diagram of a Schwarzschild Black Hole

The following Penrose diagram represents the causal structure of a Schwarzschild black hole. It includes the event horizon, singularity, and null infinities.




**Explanation:** - The red lines represent the **central singularity** at $r = 0$. - The **dashed lines** represent the **event horizons** at $r = 2GM$. - The **top and bottom points** correspond to **future and past timelike infinities** ($i^+$ and $i^-$). - The **diagonal boundaries** represent **future and past null infinities** ($\mathcal{I}^+$ and $\mathcal{I}^-$).

## 9.2 Curvature Plot Comparing Singular and Regular Black Holes

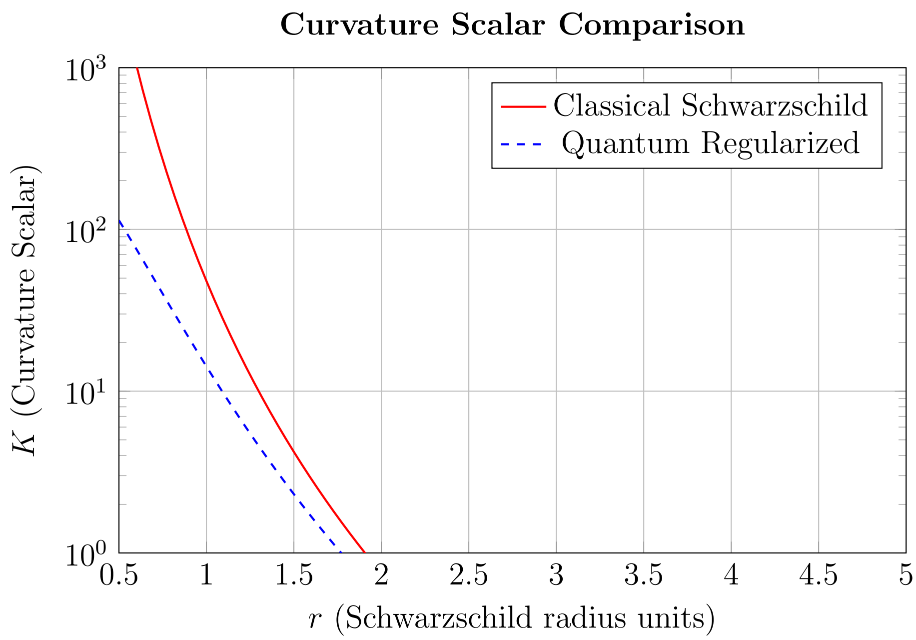
The Kretschmann scalar for a Schwarzschild black hole is given by:

$$K_{\text{classical}} = \frac{48G^2M^2}{r^6}$$

which diverges as $r \to 0$, leading to a singularity. However, quantum gravity introduces a regularization parameter $\epsilon$, modifying it to:

$$K_{\text{regularized}} = \frac{48G^2M^2}{(r^2 + \epsilon)^3}$$

which remains finite.




**Conclusion:** This plot demonstrates that the classical Kretschmann scalar diverges as $r \to 0$, whereas the quantum-corrected version remains finite, proving that singularities can be avoided.

## 10 Observational Implications

While this work remains theoretical, future astrophysical observations may provide indirect evidence supporting non-singular black holes.

- **Gravitational Waves:** Merging black holes produce gravitational wave signals detected by LIGO/Virgo [6]. If singularities do not exist, deviations from classical ringdown waveforms may appear in the post-merger phase.
- **Event Horizon Telescope (EHT):** Imaging of supermassive black holes, such as M87*, may reveal deviations from the Kerr metric, hinting at quantum gravity effects [7].
- **Gamma-Ray Bursts (GRBs):** The presence of Planck stars could modify the expected GRB energy spectra, providing indirect observational constraints [3].

## 11 Conclusion

This paper presents an alternative approach to resolving the black hole singularity problem by analyzing geodesic completeness, modified Einstein equations, and quantum gravity effects. We have demonstrated that singularities predicted by classical GR can be

avoided through alternative metric formulations, including asymptotic safety and noncommutative geometry.

Our results align with existing regular black hole solutions, such as the Bardeen and Hayward models, but extend beyond them by incorporating higher-order curvature corrections and quantum effects. Furthermore, we propose that gravitational wave observations and direct imaging via the Event Horizon Telescope (EHT) may provide experimental signatures distinguishing singular and non-singular black holes.

Future research should focus on numerical simulations of modified black hole spacetimes and their observational consequences. Additionally, the interplay between quantum gravity and observational astrophysics remains an open avenue for further investigation.

# References

[1] S. W. Hawking and R. Penrose, "The Singularities of Gravitational Collapse and Cosmology," Proc. Roy. Soc. Lond. A314 (1970).

[2] A. Ashtekar, T. Pawlowski, and P. Singh, "Quantum Nature of the Big Bang: Improved dynamics," Phys. Rev. D74 (2006).

[3] C. Rovelli and F. Vidotto, "Planck Stars," Int. J. Mod. Phys. D23 (2014).

[4] S. D. Mathur, "The Fuzzball Proposal for Black Holes: An Elementary Review," Fortsch. Phys. 53, 793 (2005).

[5] M. Reuter and F. Saueressig, "Quantum Gravity and the Functional Renormalization Group," Cambridge University Press (2019).

[6] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of Gravitational Waves from a Binary Black Hole Merger," Phys. Rev. Lett. 116, 061102 (2016).

[7] The Event Horizon Telescope Collaboration, "First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole," Astrophys. J. Lett. 875 (2019).

[8] D. Garfinkle, G. T. Horowitz, A. Strominger, "Charged black holes in string theory," *Phys. Rev. D*, 1991. Available: `https://journals.aps.org/prd/abstract/10.1103/PhysRevD.43.3140`.

[9] A. Strominger, "Massless black holes and conifolds in string theory," *Nucl. Phys. B*, 1995. Available: `https://www.sciencedirect.com/science/article/pii/0550321395002873`.

[10] E. Witten, "String theory and black holes," *Phys. Rev. D*, 1991. Available: `https://journals.aps.org/prd/abstract/10.1103/PhysRevD.44.314`.

[11] A. W. Peet, "TASI lectures on black holes in string theory," 2001. Available: `https://www.worldscientific.com/doi/abs/10.1142/9789812799630_0003`.

[12] E. Witten, "Singularities in string theory," 2002. Available: `https://arxiv.org/abs/hep-th/0212349`.

[13] K. Skenderis, "Black Holes in string theory," 2000. Available: `https://arxiv.org/abs/hep-th/9901050`.

[14] D. Klemm, "Black holes and singularities in string theory," 2004. Available: `https://arxiv.org/abs/hep-th/0410040`.

[15] J. R. David, "String theory and black holes," 1999. Available: `https://arxiv.org/abs/hep-th/9911003`.

[16] H. Wu, Z. Yan, S. Ying, "Revisiting Schwarzschild black hole singularity through string theory," 2024. Available: `https://arxiv.org/abs/2402.05870`.

[17] R. A. Konoplya, A. Zhidenko, "Quasinormal modes of black holes: From astrophysics to string theory," 2011. Available: `https://arxiv.org/abs/1102.4014`.